# STATIONARY MODES OF COMBUSTION IN A FINE-SCALE TURBULENT STREAM

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The majority of models of the turbulent combustion of gases are based mainly on intuitive concepts concerning the processes occurring in the flame. The characteristics of a turbulent flame are estimated from considerations of dimensionality and similarity. A detailed review of works on turbulent combustion is given in [1]. Problems on the calculation of the combustion rate in a turbulent stream as a proper value of the equations of heat and mass transfer and of the corresponding boundary conditions have recently been raised. Here too one must rest on assumptions of a semiempirical nature, which in large measure is connected with the inadequate level of development of turbulence theory. In the present work the equation of propagation of the zone of chemical reactions in the stream is averaged statistically by analogy with studies of turbulent flows. Correct averaging is possible at scales of hydrodynamic disturbances smaller than the flame thickness (fine-scale turbulence). The temperature pulsations are related with the size of the heat flux using the theory of mixing lengths. The main influence is specific to effects arising during averaging of the heat release function. Two stationary modes, distinguished by the normal propagation velocity $\omega_1$, are isolated within the framework of the Cauchy problem with a given initial mixture temperature and zero heat flux in the burned gas. A heat conduction mode occurs with a stream velocity $\omega > \omega_1$ and an induction mode with $\omega < \omega_1$. An expression is found for $\omega_1$ which reflects the principal effects in the flame and which in the limit coincides with the equation of Zel'dovich and Frank-Kamenetskii for a laminar flame. In those cases when the distorting effect of the heat release function is small, the turbulence affects the combustion rate through mechanisms of intensification of transport processes.

1. Bearing in mind the case $\mathrm{Le} = D/\varkappa = 1$, let us examine only the energy equation. Furthermore we will use the notation of [3]. The small scales of the disturbances make it possible to consider the one-dimensional system

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial x} = \varkappa\frac{\partial^2 T}{\partial x^2} + \Phi(T) \tag{1.1}$$

Here the actual values are related with the average values $\langle w\rangle$, $\langle T\rangle$ and the pulsation components $w'$, $T'$ by the equations

$$T = \langle T\rangle + T', \qquad w = \langle w\rangle + w' \tag{1.2}$$

For the relation between the hydrodynamic and thermal fields one can draw on the theory of the mixing length of Prandtl

$$T' = l_1\partial\langle T\rangle / \partial x \tag{1.3}$$

Averaging (1.1) with allowance for (1.2) and (1.3) and using Reynolds' rules gives

$$\langle w\rangle\frac{\partial\langle T\rangle}{\partial x} = (\varkappa + l_1 w')\frac{\partial^2\langle T\rangle}{\partial x^2} + \langle\Phi(T)\rangle \tag{1.4}$$

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The influence of irregular convective heat transfer on the coefficient of molecular thermal diffusivity has always been taken into account, but the distortion of the heat release function and the influence of this effect on the characteristics of the flame has been taken into account only in recent works (see [1-3] and the literature cited there).

In [3] a study was made of an analog of Eq. (1.4) with the corresponding boundary conditions composing the problem on the proper value. When the heat release function for highly exothermic flames (the temperature of the burned gas is more than twice as high as the temperature of the cold mixture) is cut off at the cold boundary the proper value of the problem, corresponding to the "normal" velocity of the turbulent value, exists only when $l_1 \leq l_*$. A more detailed study shows that this condition is not violated for fine-scale turbulence.

The necessity of cutting off the heat release function at the cold boundary is not always justified in technical settings. Frequently the thermal conditions at the inlet of the combustion system are such that the characteristic chemical time at the cold end is comparable to and even less than the time of the process, so that the intermediate asymptotic form of the solution of the problem for a normal flame proves to be incorrect.

The stationary modes of existence of a flame in a laminar stream, when the chemical reaction rate at the initial temperature cannot be neglected, are described in [4, 5].

We will investigate the effect of turbulence on such flames starting from the equation (we will omit the averaging brackets)

$$\frac{dp}{du} = \frac{\Phi(u, p)}{p} - \omega \tag{1.5}$$

with the initial (in the sense of the Cauchy problem) condition

$$p = 0, \quad u = 0 \tag{1.6}$$

The dimensionless form of Eq. (1.5) is related to Eq. (1.4) by the equations

$$p = -\frac{\partial u}{\partial \xi}, \quad u = \frac{T_+ - T}{T_+ - T_-}, \quad \xi = \frac{x}{[(\varkappa + \varkappa_1)\tau_+]^{1/2}} = \frac{x}{x_+}$$

$$\tau_+ = \rho^{1-n} z^{-1} \exp(E / RT_+)$$

$$\theta_0 = E(T_+ - T_-) / RT_+^2$$

$$\omega = w(\varkappa + \varkappa_1)^{-1/2} \tau_+^{1/2}, \qquad \varkappa_1 = l_1 w',$$

$$\sigma = 1 - T_- / T_+ = Q / cT_+$$

The average dimensionless heat release function for the first-order reaction has the form [3]

$$\begin{aligned} 2\Phi(u, p) = (u + Fp) \exp - \left[\frac{\theta_0 (u + Fp)}{1 - \sigma(u + Fp)}\right] + \\ + (u - Fp) \exp\left[-\frac{\theta_0 (u - Fp)}{1 - \sigma(u - Fp)}\right] \end{aligned} \tag{1.7}$$

where the parameter F characterizes the relative scale of the turbulent pulsations

$$F = l_1 / x_+ \tag{1.8}$$

The initial condition (1.6) means the absence of heat fluxes after the flame reaches the maximum temperature.

The total thermal diffusivity coefficient $\varkappa + l_1 w'$ enters into the scaling variables, i.e., the dependence of $l_1 w'$ on the parameters of the main stream is actually neglected. Such a region of constant $\varkappa_1$ is realized for flows in pipes, when Re > $10^4$ [6]. In this case one can isolate the effect of distortion of the average heat release function in pure form, separating the well-known Damköhler effect of an increase in the intensity of transport processes in a turbulent stream.

2. The problem (1.5)-(1.7) was analyzed on an electronic computer. A study was made of the function $u(\xi, \omega, \sigma, F, \theta_0)$ and the characteristics connected with it: L is the length of the preflame zone, or the distance to the point of maximum reaction rate reckoned from the cold end, and $p_1$ is the heat flux at the cold boundary. The ranges of variation of the parameters of the problem are: $0 \leq \omega \leq 10$, $0 \leq F \leq 10$, $0 \leq \sigma \leq 0.8$, $1 \leq \theta_0 \leq 10$. The principal results are presented in the graphs. Although calculations are given only for a small selection of parameters, nevertheless all the important aspects of the qualitative pattern of the phenomenon are reflected in them.

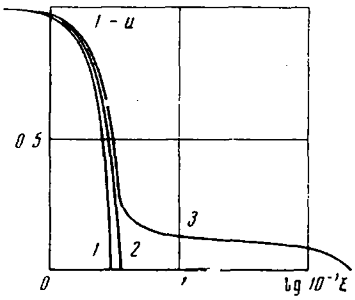


Fig. 1

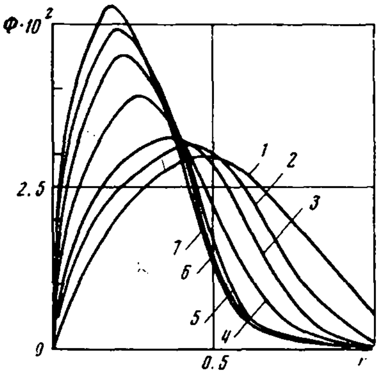


Fig. 2

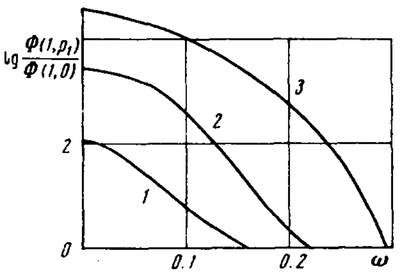


Fig. 3

The behavior of the temperature curves for different $\omega$ is shown in Fig. 1, with $\theta_0 = 10$, $\sigma = 0.2$, and $F = 6$. The numbers 1, 2, and 3 correspond to $\omega = 0.1$, 0.2, and 0.3. For small $\omega$ the region of high heat fluxes practically coincides with the length of the flame, while at rather high $\omega$ the heat fluxes are small on a considerable length of flame. In the first case the original mixture is prepared for combustion through conductive heat transfer (heat-conduction mode), while at large $\omega$ the cold gas is warmed by the heat liberated in the course of the chemical reaction (induction mode).

The condition separating the two modes is the equality of the stream velocity $\omega$ with the normal velocity of flame propagation $\omega_1$, which can be determined as the proper value of the problem (1.5)-(1.7). When $\omega < \omega_1$ the flame travels against the stream, contracting the size of the warming zone. The heat flux into the cold mixture grows accordingly, which has a stabilizing effect on the flame motion. If $\omega > \omega_1$ the stream carries the flame down along the flow until the time of movement of a portion of gas becomes insufficient for the ignition of this portion.

We can apply the views developed in [5] for the laminar case at $F = 0$ to the problem of a fine-scale flame. Here it is essential that $\omega$, being an arbitrary parameter of the problem, determines the convective part of the heat exchange.

For a fixed $\omega$ and constant kinetic parameters the stationary occurrence of the process is possible for a single value of the heat flux into the cold mixture. Thus, the temperature gradient at the cold boundary and in the entire region is related to the value of $\omega$.

If $F = 0$ the heat release function $\Phi(u)$ does not vary with $\omega$ and has a maximum at $u \approx 0.1$. When $F > 0$ the dependence of the average reaction rate on the temperature gradient gives rise to its dependence on $\omega$. One can see from Fig. 2 that with a drop in $\omega$ the reaction rate maximum is broadened, approaching the cold end. Curves 1-7 correspond to $\omega = 0.1$, 0.2, 0.3, 0.6, 1, 1.5, 2. At the very edge $u = 1$ the heat release function does not equal zero for small $\omega$. This is possible because of the large temperature gradient. With an increase in the stream velocity the gradient decreases and its distorting effect on the heat release function disappears.

In Fig. 3 is given the dependence of the logarithm of the ratio of the turbulent $\Phi(1, p_1)$ and laminar $\Phi(1,0)$ reaction rate functions at the cold boundary on $\omega$ for different values characterizing the degree of turbulence. The values $F = 3$, 6, 10 correspond to the curves 1, 2, 3. At certain $\omega$, depending on F, the reaction rates in the average turbulent and laminar streams are comparable. This value of $\omega$ corresponds to the normal propagation velocity of the flame.

The changes in the length of the preflame zone as a function of $\omega$ are shown in Fig. 4. The family of curves A was obtained for $\theta_0 = 6$, $\sigma = 0.8$ $(\theta_0/(1-\sigma) = 30)$, B for $\theta_0 = 10$, $\sigma = 0.2$ $(\theta_0/(1-\sigma) = 12.5)$, and C for $\theta_0 = 6$, $\sigma = 0.2$ $(\theta_0/(1-\sigma) = 7.5)$. In this figure the numbers 1, 2, 3, 4 are introduced to mark the curves corresponding to $F = 0$, 3, 6, 10. It is seen that at a certain $\omega$ one stationary combustion mode with small heating zones is replaced by another mode with a preflame zone several orders of magnitude larger.

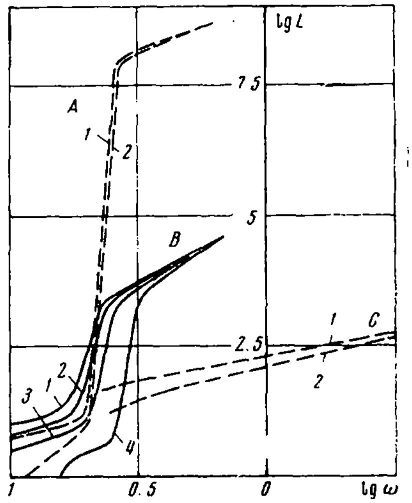


Fig. 4

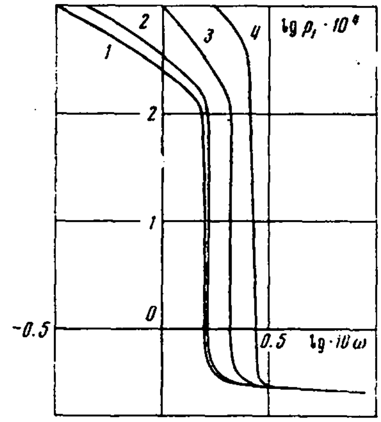


Fig. 5

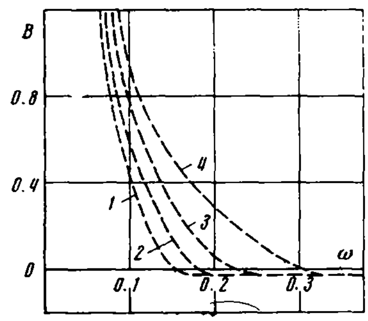


Fig. 6

This transition is accomplished at different $\omega_1$ as a function of F. The depth and sharpness of the transition depend on the value of $\theta_0/(1-\sigma)$. At small values of $\theta_0/(1-\sigma)$ the transition region is smeared out. At large values of $\theta_0/(1-\sigma)$ the curves corresponding to different F almost run together for all values of $\omega$, i.e., the distortion of the heat release function by fine-scale turbulence is insignificant for large activation energies E or small values of $Q/cT_+$.

The closing up of the $L(\omega)$ curves when $\omega > \omega_1$ indicates that in this mode the turbulence ceases to affect the flame characteristics, since the heat fluxes are small in almost the entire region of occurrence of the chemical reactions and the almost isothermic gas mixture does not undergo temperature disturbances.

Figure 5 gives a representation of the behavior of the $p_1$ curves as a function of $\omega$ for different F. As in the preceding case, the values F=0, 3, 6, 10 correspond to the curves 1, 2, 3, 4. Here too one can illustrate all the above remarks concerning the properties of the combustion process in a turbulent stream.

Finally, the values of

$$B = \left[\int_0^\infty \Phi\, d\xi - \omega\right] / \omega^{-1}$$

which characterizes as a function of $\omega$ the contribution of the conductive heat flux compared with the convective heat removal are plotted in Fig. 6. Here the numbers 1, 2, 3, 4 correspond to curves with F=0, 3, 6, 10. For $\omega < \omega_1$ this value is large and the heat is carried mainly by the conductive mechanism, while for $\omega > \omega_1$ the total heat release is almost fully compensated for by the convective flux.

3. The existence of two stationary combustion modes in a stream makes it possible to estimate the normal velocity of a turbulent flame. When $\omega < \omega_1$ we can neglect the convective term in Eq. (1.5), leaving the initial condition (1.6) as before

$$dp^2 / du = 2\Phi(u,\ p) \tag{3.1}$$

Assuming that the temperature disturbances are not very large, we expand the average reaction rate function with respect to the small value $u'/u = Fp/u$. It is easy to verify that the first terms of the expansion are written in the form

$$\Phi(u, p) = \Phi(u) + \frac{F^2 p^2}{u^2} \frac{\partial^2 \Phi}{\partial u^2} \tag{3.2}$$

The linear term of the expansion is absent because of the symmetry of the function. For simplicity the value $F^2/u^2$ was replaced by $F^2/\langle u_1\rangle^2$, where $\langle u_1\rangle$ is some mean value between zero and one.

For a sufficiently large $\theta_0$ an estimate of the heat flux in the heat-conduction mode at the cold end, where u = 1, gives

$$p_1^2 = (1 + 2\alpha_1 F^2 / \langle u_1^2 \rangle)\left[ 2\int_0^1 \Phi(u)\, du + \Phi_1^2 F^2 / \langle u_1 \rangle^2 \right] \tag{3.3}$$

where

$$\alpha_1 = \left(\frac{\theta_0}{1-\sigma} - 1\right) \exp\left(-\frac{\theta_0}{1-\sigma}\right)$$

$$\Phi_1 = \exp\left(-\frac{\theta_0}{1-\sigma}\right)$$

With an increase in the velocity of the impinging stream the role of convective heat transfer grows and at some $\omega = \omega_1$ it becomes comparable with the conductive heat exchange.

This relationship of the heat fluxes at the cold boundary u = 1 is written in the form

$$p_1 = \omega_1 \tag{3.4}$$

which in conjunction with (3.3) gives the flame velocity in the transitional mode

$$\omega_1^2 = \left(1 + \frac{2\alpha_1 F^2}{\langle u_1 \rangle^2}\right)\left(2\int_0^1 \Phi(u)\, du + \frac{\Phi_1^2 F^2}{\langle u_1 \rangle^2}\right) \tag{3.5}$$

The value F = 0 determines the laminar velocity of normal flame propagation

$$\omega_0 = 2\int_0^1 \Phi(u)\, du$$

The results obtained are identical to the well-known equation of Zel'dovich and Frank-Kamenetskii, which should be expected since their procedure for obtaining $\omega_0$ was formally repeated. But in the present case the fundamental equation (1.5) was broken down not on the basis of the spatial structure of the normal flame, but of the relationship between the different mechanisms of heat transfer as a function of $\omega$.

Equation (3.5) reflects the principal aspects of the process qualitatively correctly. In particular, at large values of $\theta_0/(1-\sigma)$ we have $\alpha_1 \approx \Phi_1 \ll 1$, therefore $\omega_1 \to \omega_0$ and the other characteristics of the flame are also close to laminar. In calculating $\omega_1$ the value $\langle u_1 \rangle$ was taken at the point where the maximum heat release is reached, $\langle u_1 \rangle \approx 0.1$. The estimate of the normal combustion rate used here gives an understated value (by about 30%) compared with the results of a numerical calculation, which is quite explainable since the assumption that the temperature pulsations are small, which is essential in the derivation of the equation, is not always satisfied in the computer calculations.

It must be stipulated that even in the case where the influence of the factor of distortion of the heat function is very small, for example $\theta_0/(1-\sigma) \gg 1$, turbulence continues to play a role in flame propagation through mechanisms of intensification of transport processes. In the physical variables this effect is reflected in the relationship between the normal turublent and normal laminar velocities

$$w_T = w_n \left[(\varkappa + l_1 w') / \varkappa\right]^{1/2}$$

In conclusion we note that the boundary condition at the cold boundary used in [4] provides for the stationary course of the process at a single value of the maximum flame temperature $T_+$. In the present work, as in [5], the temperature $T_+$ is uniquely determined through the initial flame temperature $T_-$ and the given thermokinetic characteristics of the gas. In this case the stationary mode determines the single value of the heat flux in the original mixture. Its value is easy to estimate in particular cases. In the heat-conduction mode it is determined from the solution of the equation

$$dp / du = \Phi(u, p) / p$$

with the condition u = 0, p = 0 at the point u = 1; in the extreme induction mode the stationary gradient must be sought for as a root of the transcendental equation $p = \Phi(1, p)/\omega$.

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